PRECIPITATION OF A UNIPOLAR-CHARGED AEROSOL ON AN ARRAY OF GROUNDED CONDUCTORS

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An approximate method for calculating precipitation of a unipolar-charged aerosol on a unit conductor of spherical or cylindrical shape placed at a certain height above a grounded plane surface in a uniform electric field, and also on arrays of grounded conductors, is described. Experimental results are reported.

The precipitation of suspended unipolar-charged particles of charge q, mobility k, and calculated concentration n_0 onto a grounded conductor of surface charge density σ and electric field strength at the surface E_a, is given by the formula

$$N = n_0 k q E_a$$

where N is the number of particles precipitated on a unit surface area of the conductor in 1 sec; the gravitational and other forces acting on the particle are here assumed much smaller than the electric field, so that they can be safely left out of account.

The average field strength E_a at the surface of the body is determined by the charge Q on the body. The problem thereby becomes reducible to determination of the charges on each of the conductors comprising the array.

The charge induced on a conducting grounded sphere immersed in the earth's electric field

$$Q_1 = 4\pi\varepsilon_0 \varepsilon a E_0 H = K$$

where a is the radius of the sphere, ε_0 is the permittivity, ε is the relative permittivity (dielectric constant) of the medium, E_0 is the strength of the undisturbed field, and H is the height of the sphere above ground.

The average field strength at the surface of the sphere

$$E_1 = Q_1 / 4\pi\varepsilon_0 \varepsilon a^2 = E_0 H / a \tag{1}$$

We do take into account the effect of charges induced on the earth's surface. We resort to the method of images to do this, i.e., we replace the earth by a fictitious sphere of equal size placed symmetrical with respect to the earth's surface and having a charge of the same magnitude and opposed sign. According to the superposition principle, the potential of the bounded sphere (neglecting "bias," i.e., when $a/H \ll 1$)

$$\varphi = E_0 H - \frac{Q_2}{4\pi\varepsilon_0\varepsilon a} + \frac{Q_2}{8\pi\varepsilon_0\varepsilon H} = 0$$

Hence

$$Q_2 = K\left(1 - \frac{a}{2H}\right)^{-1} \tag{2}$$

We now turn to the case of two grounded spheres at height H, neglecting bias, i.e., assuming $a/b \ll 1$, where b is the distance separating the two spheres. As a consequence of symmetry, the induced charges on

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the two spheres are equal. The condition that the potential on the surface of one of them be zero is

$$\varphi = E_0 H - \frac{Q_3}{L}\left(\frac{1}{a} - \frac{1}{2H} + \frac{1}{b} - \frac{1}{\sqrt{b^2+4H^2}}\right) = 0 \quad (L = 4\pi\varepsilon_0\varepsilon)$$

Hence

$$Q_3 = K\left[\left(1 - \frac{a}{2H}\right) + \frac{a}{b}\left(1 - \frac{1}{\sqrt{1+4H^2/b^2}}\right)\right]^{-1} \tag{3}$$

In the case where n > 2 spheres are present, their charges are unequal, and the calculations boil down to solving a system of first-degree equations with the number of equations either n/2 or n/2 + 1. The numerical solutions are obtained on a computer. These show that, in the case of spheres situated closer to the middle of the row, the charges are smaller than on the extreme outlying spheres, and that the charges decrease with increasing n, other things being equal.

In addition to the n = 2 case, the charges on all the spheres turn out to be equal in the limit n = ∞. This case is not only a limiting case, but is of independent interest in its own right: it can be treated, for instance, as an approximate model of a large number of products moving on a suspended conveyor belt in slow advance through an electrostatic paint-spraying zone.

When n = ∞, the zero potential condition on the surface of one of the spheres is

$$\varphi = E_0 H - \frac{Q_4}{L}\left(\frac{1}{a} - \frac{1}{2H} + \frac{2}{b} - \frac{2}{\sqrt{b_2+4H^2}} + \right.$$
$$\left. \cdots + \frac{2}{n^2b^2} - \frac{2}{\sqrt{n^2b^2+4H^2}} + \cdots\right) = 0$$

Hence, after some straight forward transformations,

$$Q_4 = K\left[\left(1 - \frac{a}{2H}\right) + \frac{2a\Gamma}{b}\right]^{-1}, \quad \Gamma = \sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{\sqrt{n^2+c}}\right), \quad c = 4H^2/b^2 \tag{4}$$

Enlisting the aid of the integral convergence criterion, we can estimate the degree of accuracy Γ in the calculations when the number of terms in the series is n: the residue

$$R_n = \sum_{n+1}^{\infty}\left(\frac{1}{n} - \frac{1}{\sqrt{n^2+c}}\right)$$

satisfies the inequality

$$\ln\frac{1+\sqrt{1+c/(n+1)^2}}{2} < R_n < \ln\frac{1+\sqrt{1+c/n^2}}{2}$$

Equations (3) and (4) define the top and bottom limiting values of the charge on the sphere. The numerical calculations show that when there is a finite number of spheres present, other conditions being equal, the charge Q on any sphere will satisfy the inequality $Q_3 > Q > Q_4$.

This is also inferred directly from equations of type (4): each increase by two in the number of spheres present, i.e., addition of two pairs of symmetrical and equal charges, results in a lowering of the charge on the central sphere, i.e., this charge tends monotonically to some limit as $n \to \infty$. Under the conditions assigned in the experiments (see below), we have $Q_3/Q_4 = 3.85$, i.e., the range of Q values for different n is relatively restricted, and Eqs. (3) and (4) are valid for approximate estimates and as checks on the numerical calculations.

By utilizing the same method, we can derive similar formulas for systems or arrays consisting of cylindrical conductors of infinite extent. The induced Q' are referable to a unit length of the cylindrical conductor in that array. In the case of a single cylindrical conductor in the uniform field of the earth at a height H, with attention given to the charges induced on the surface of the earth,

$$Q_2' = \frac{K}{2a}\left(\ln\frac{2H}{a}\right)^{-1} \tag{5}$$

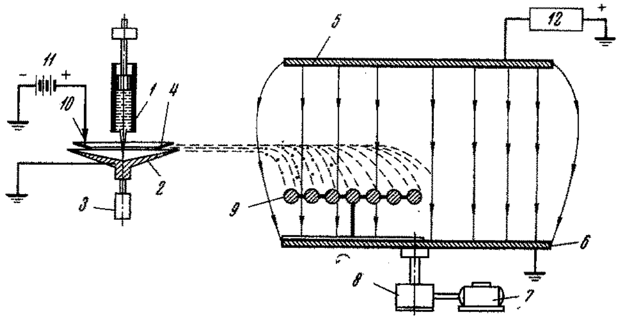

Fig. 1

In the case of two conductors

$$Q_3' = \frac{K}{2a}\left\{\ln\left[\frac{2H}{a}\left(1+\frac{4H^2}{b^2}\right)^{1/2}\right]\right\}^{-1} \tag{6}$$

In the case of an infinitely large number of conductors (which can be treated, for example, as a model of agricultural crops sowed in row drills)

$$Q_4' = \frac{K}{2a}\left(\ln\frac{2H\Pi}{a}\right)^{-1}, \quad \Pi = \prod_{n=1}^{\infty}\left(1+\frac{c}{n^2}\right) \tag{7}$$

According to [1]

$$\prod_{n=1}^{\infty}\left(1+\frac{c}{n^2}\right) = \frac{\operatorname{sh}\pi\sqrt{c}}{\pi\sqrt{c}} = \frac{\operatorname{sh}(2\pi H/b)}{2\pi H/b}$$

i.e.,

$$Q_4' = \frac{K}{2a}\left[\ln\left(\frac{b}{\pi a}\operatorname{sh}\frac{2\pi H}{b}\right)\right]^{-1} \tag{8}$$

As in the case of spheres, Eqs. (6) and (8) determine the top and bottom limiting values of the linear density of the charge Q' induced on the cylinders. The numerical calculations reveal that when the number of cylinders is finite and other conditions are the same $Q'_4 < Q' < Q'_3$; as in the case of spheres, this is also directly evident from formulas of type (7). Under the conditions stipulated in the experiments, $Q'_3/Q'_4 = 4.27$, i.e., the range of Q' values at different n is relatively restricted.

Precipitation of a unipolar-charged aerosol in a relatively uniform electric field onto grounded conductors of spherical or cylindrical shape was studied as part of an experimental verification of these results. The aerosol was charged by an inductive method, with the aid of a conical electrifying spray device [2].

The experimental arrangement is shown diagramatically in Fig. 1. The liquid is a mixture of glycerin (60%) and water (40%) with fluorescein (0.4%) added dropwise from a syringe 1 to the cone 2, which is rotated by an electric motor 3 at 2000 rpm. The liquid flowrate was 0.075 cm³/sec. A metal ring 4, rotating in unison with the cone and functioning as an electrode, was mounted on insulators on the cone; the clearance between the metal ring and the cone was 0.5 cm. A voltage of positive polarity (1100 V) was supplied to the ring-electrode via slip rings 10 from "Molniya" type storage batteries connected in series.

As a result, negative electrical charges were induced on the surface of the liquid film moving over the cone in response to centrifugal forces (positive charges leaked off through the grounded cone 2 to the ground). The charged liquid film broke up on the edge of the rotating cone to form droplets which were then thrown off the periphery of the cone. The bulk of these droplets, possessing a high initial velocity, landed on the inner walls of the stationary grounded body (not shown in the diagram), but some of them were ejected through a port in the body, and were carried off by a stream of air from the guide fan (also not shown on the diagram), and transported to a region of approximately constant electric field set up between the electrode 5 and the grounded turntable 6, the latter driven to rotate at 15 rpm by the electric motor 7 working through a reducing gearset 8 (high voltage of negative polarity, V = 20-30 kV, is supplied to the electrode 5 from the AII-70 high-voltage generator 12). The distance between the electrode 5 and grounded turntable 6 was set at 80 cm. Spherical or cylindrical conductors 9, on which the charged droplets settled, were mounted on the rotating turntable 6 at a height of 12 cm. The conductors were precoated with a thin layer of silicone on which the spreading coefficient of droplets of the different sizes of the water–glycerine mixture was approximately constant [3]. At 45-60% air humidity, the diameter of the settled droplets (assuming lenticular shape) remained practically constant throughout the experiment [3].

A supply of charged aerosol was provided in the experiment, and the rotating turntable was used to drive the conductors through a "shower" of precipitating droplets in the uniform electric field.

The conductors with droplets settled on them were examined in ultraviolet light under an MBS-1 microscope; the droplets settled on the top and bottom surfaces of the conductor were counted and sized, with the amount of area inspected taken into account.

In order to determine the total charge on the droplets settling in the region of constant electric field between the turntable 6 and the electrode 5, a grounded metal plate extending in 5000 cm^2 in area was mounted on the turntable, and silicone-coated microscope slides were distributed over the turntable. The charging current I passing through the metal plate to ground, and due to the precipitation of the charged droplets, was measured, as well as the total quantity of liquid settled on the metal plate and the size distribution of the settled droplets. The values obtained were I = 0.005 μA and Q = 0.0014 and cm^3/sec.

In order to determine the individual charges on droplets of different sizes from results of measurements of the total charge on the droplets, use was made of the fact that the individual charges on the droplets are proportional to the radius of the droplets in the inductive method of charging [4]. With the aid of the system of equations

$$I = N_1 q_1 + N_2 q_2 + \ldots + N_m q_m$$
$$q_1 / q_2 = r_1 / r_2, \quad q_1 / q_3 = r_1 / r_3, \quad \ldots, \quad q_1 / q_m = r_1 / r_m$$

the values of the charges q on droplets of different sizes were computed. Here we cite the distribution of the the number of droplets n in percentages of the total number of droplets N, and also the distribution of the corresponding charges q [CGSE units] as a function of the droplet radius r in microns

$r = 5$	9	14	19	24	30	37	45	53
$n = 20$	18	26	16	8.9	6.2	3.2	1.1	0.02
$q \cdot 10^5 = 2.2$	4.0	6.2	8.2	10.6	13.3	17	20	23

Results of experiments conducted with a single sphere (a = 1.3 cm, H = 12 cm) are plotted in Fig. 2a (curve 1). The droplet radius r is plotted as abscissa, the degree of nonuniformity in the deposition of the droplets $\varkappa = m_+/m_-$ is plotted as ordinate, and here m_+ is the number of droplets of radius r settled out in the neighborhood of the top pole of the sphere per unit surface area, and m_- is the counterpart number for the bottom pole of the sphere. The experimental results are indicated by the hollow circles, while the dashed curve (curve 1) shows the theoretical dependence

$$\varkappa = \frac{qE_2 + Mg}{qE_2 - Mg} \tag{9}$$

representing the ratio of the forces acting on droplets settling out on the pole (precipitation occurring in response to the sum of the electrical force qE_2 and the gravitational force Mg, where M is the droplet mass) to those forces acting on droplets settling out on the bottom pole (the difference in these forces acts). When the concentration of droplets in the neighborhood of the sphere is constant, and when there is a linear relationship between the forces acting on the droplet and the velocity at which the droplet moves (Stokes law), the ratio of forces $\varkappa$ becomes equal to the ratio of the corresponding streams of precipitating droplets. The quantity E_2 is the field strength at the surface of the sphere and was calculated on the basis of Eqs. (1) and (2).

Figure 2b represents the theoretical and experimental values of the ratio of the number of droplets settled on the top pole of the sphere in the presence of electrification and in its absence, $p = m_{e+}/m_+$, where m_{e+} is the number of droplets of radius r settled out in the neighborhood of the top pole of the sphere per unit surface area in electrification, and m_+ is the same in the absence of electrification, all other conditions being the same. The experimental P values are marked by hollow circles. The dashed curve (curve 1) represents the dependence $P = f(r)$ calculated on the basis of the formula

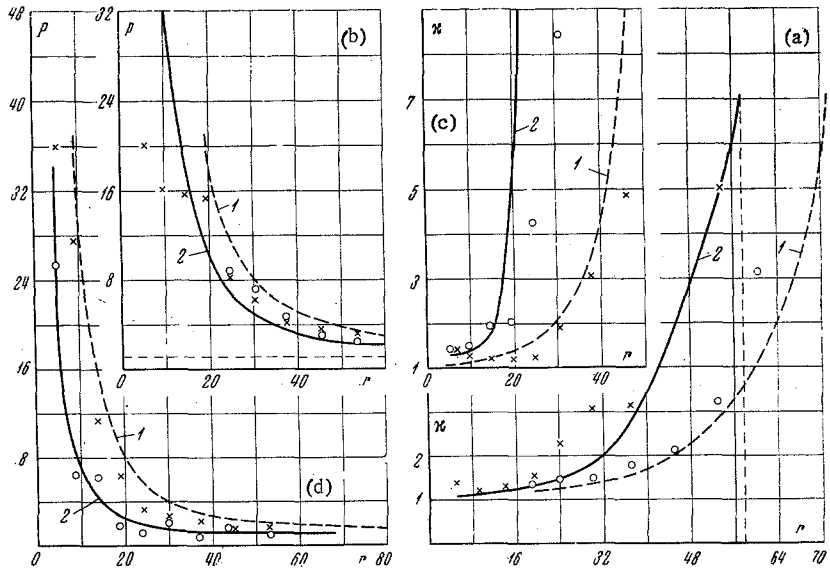

Fig. 2

$$P = \frac{qE_2 + Mg}{Mg} \tag{10}$$

with the aid of Eqs. (1) and (2).

An array consisting of a row of seven spheres was used in the subsequent experiments. The crosses in Fig. 2a indicate the experimental values of $\varkappa$ for the middle sphere in that array (a = 1.3 cm, b = 4 cm, H = 12 cm). The theoretically predicted values (continuous curve, curve 2) were calculated with the aid of a a digital computer solution of the corresponding system of four equations. The P values for that system are plotted in Fig. 2b, where the experimental data are represented by crosses, and the theoretically predicted data are represented by the continuous curve (curve 2).

Similar experiments using three spheres were also performed, making use of the solution of the corresponding system of two equations, and experiments using a single sphere were staged, with the values a = 0.7 cm, a = 2.2 cm, H = 6 cm, V = 30 kV. Agreement between theoretically predicted and empirical values of $\varkappa$ and P was also satisfactory in these last experiments.

We then proceeded to experiments using steel grounded cylinders of radius a = 1 cm and length l = 12 cm. These cylinders were set at height H = 12 cm. The measurements of the precipitated droplets were taken at midcylinder, in the vicinity of the top and bottom generatrices of the cylinder.

Experimental $\varkappa$ values are plotted (as crosses) for one cylinder in Fig. 2c, and also for the middle cylinder in an array of seven situated parallel to each other with pitch b = 4 cm and in the horizontal plane H = 12 cm (hollow circles). The dashed curve (curve 1) indicates the theoretical curve $\varkappa = f(r)$, calculated on the basis of Eq. (9) with the aid of Eq. (5) for a single cylinder. The continuous curve (curve 2) corresponds to the theoretical dependence $\varkappa = f(r)$ for seven cylinders and was obtained by numerical solution of the corresponding system of four equations on a digital computer.

The corresponding experimental and theoretical values of P are indicated in Fig. 2d. The crosses mark experimental results obtained for one cylinder, the hollow circles experimental results for an array of seven cylinders. The dashed curve 1 corresponds to the theoretical depencence P = f(r) for a single cylinder, as calculated on the basis of Eqs. (5) and (10). The continuous curve 2 represents the theoretical dependence P = f(r) for the middle sphere in an array of seven cylinders, and was obtained by digital-computer numerical calculation of the corresponding system of four equations.

The results of similar experiments and computations using three parallel cylinders (a = 1 cm, l = 12 cm, H = 12 cm, b = 4 cm) were also found to be in satisfactory agreement.

The set of results appearing in Fig. 2 and the results of the supplementary experiments mentioned attest to the approximate degree of agreement between experiments and calculations at $a/H \leq 0.116$ and $a/b \leq 0.325$.

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